Lassonde Faculty of Engineering
EECS
MATH1090A. Problem Set No1
Posted: Sept. 19, 2017

Due: Oct. 4, 2017, by 2:30pm; in the course assignment box.

It is worth remembering (quoted from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (5 MARKS) Prove by analysing formula-calculations that $((\neg\bot))$ is not a well-formed-formula.

2. Prove that no wff can be empty; specifically *prove that it must contain at least one atomic formula as a substring.*

Required methodology: By analysing formula-calculations, or by induction on formulas.

3. (1 MARK) Prove that $\Big(\big(r \to ((\neg(p \to q)) \land p)\big) \to \bot\Big)$ *is* a wff.

4. (6 MARKS) Recall that a *schema* is a tautology iff ***all*** its *instances* are tautologies. Thus,

A schema is *not* a tautology **iff** *it has an instance that is not.*

Which of the following six schemata are tautologies? Show the whole process that led to your answers, *including truth tables or* equivalent short cuts, *and words of explanation*.

I note that in the six sub-questions below I am *not* always using all the formally necessary brackets. It is your task to correctly insert any missing brackets before you tackle the question for each formula.

- $A \vee B \equiv A \wedge B \equiv A \equiv B$
- $A \vee B \rightarrow A \rightarrow B$
- $A \wedge B \rightarrow A \vee B$
- $A \rightarrow B \equiv \neg B \rightarrow \neg A$
- $A \wedge B \rightarrow (A \equiv B)$
- $A \wedge (B \equiv C) \equiv A \wedge B \equiv A \wedge C$

5. (5 MARKS) Use Induction on the number n to prove

$$A_1, A_2, \ldots, A_n \models_{taut} B \text{ iff } \models_{taut} A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_n \rightarrow B$$

Hint. There are two directions to prove!

6. (3 MARKS) Prove that, for any formulas A, B and C, we have

$$\text{If } A, B \models_{taut} \bot, \text{then also } A, B \models_{taut} C$$

7. (5 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.

In order to show that a tautological implication that involves *meta*-variables for formulae —i.e., it is a schema— is *incorrect* you *must* consider an *instance* (i.e., a special case with specific formulae) that *is* incorrect (since some other special cases might work).

Show the whole process that led to each of your answers.

- $p \vee q \models_{\text{taut}} p$
- $A \models_{\text{taut}} A \vee B$
- $\neg A \wedge A \models_{\text{taut}} B$
- $\neg A \vee A \models_{\text{taut}} B$
- $B, B \rightarrow A \models_{\text{taut}} A$

8. (6 MARKS) Write down the most simplified result of the following substitutions, *whenever the requested substitution makes sense*. Whenever a requested substitution does not make sense, explain exactly why it does not.

Show the whole process that led to each of your answers in each case.

Remember the priorities of the various connectives as well as that of the meta-expression "[$\mathbf{p}$:= ...]"! The following formulae have not been written with all the formally required brackets.

- $p \to \top[p := \top]$
- $p \vee q \wedge r[q' := \bot]$
- $p \vee q \wedge r[q := A]$ (where A is some formula)
- $p \vee (q \to p)[p := r]$
- $(p \vee q)[p := \mathbf{t}]$
- $(p \vee q)[(p \vee q) := r]$