Lassonde School of Engineering

Dept. of EECS
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MATH1090 B. Problem Set No2

Posted: Oct. 9, 2019

Due: Oct. 30, 2019; by 2:00pm, in the course assignment box.

In this problem set and onwards, $\mathbf{p}, \mathbf{q}, \mathbf{r}'$ etc., are *metavariables* that stand for *actual* Boolean variables. As such, it is possible that, say, $\mathbf{p}$ and $\mathbf{q}$ stand for the same actual variable in some line of reasoning.

1. (8 MARKS)

Prove that

$$\vdash A \equiv A$$

in **two different ways** that do **not** use the "trick" of a Leibniz variable $\mathbf{p}$ that does not appear in A.

2. (5 MARKS) *True* or *False* **and WHY**?

The following two statements —(1) and (2)— are equivalent

$$\Gamma \vdash A \text{ iff } \Gamma \vdash B \tag{1}$$

$$\Gamma \vdash A \equiv B \tag{2}$$

3. (5 MARKS) We have learnt that $\Gamma \vdash A \wedge B$ implies that $\Gamma \vdash A$ **AND** $\Gamma \vdash B$.

Is (1) below *True* or *False* **and WHY**?

$$\Gamma \vdash A \vee B \text{ implies that } \Gamma \vdash A \textbf{ OR } \Gamma \vdash B \tag{1}$$

Caution. If a proof style is explicitly **required** in what follows, then any other style used gets 0 marks regardless of its correctness.

4. (5 MARKS) Prove **Equationally** that

$$A, \neg A \vdash \bot$$

directly, **without** using the derived rule CUT in **any** of its special forms.

5. (4 MARKS) Prove **Equationally** that $A \vdash B \rightarrow A$.

6. (4 MARKS) Prove **Equationally** that $A \vee B \vdash \neg B \rightarrow A$.

7. (5 MARKS) Prove that $A \rightarrow B \vdash A \vee C \rightarrow B \vee C$.

8. Prove that $A \rightarrow B, A \rightarrow C \vdash A \rightarrow B \wedge C$.

Do **two** proofs:

- (3 MARKS) One **with** the Deduction theorem (and a Hilbert-style proof).
- (5 MARKS) One Equational, **WITHOUT** using the Deduction theorem.