Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

MATH1090 A. Problem Set No2

Posted: Oct. 9, 2020

Due: Oct. 30, 2020; by 2:00pm, in eClass, "Assignment #2"

Q: How do I submit?

A:

(1) **Submission must be ONLY ONE file**

(2) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

In this problem set and onwards, $\mathbf{p}, \mathbf{q}, \mathbf{r}'$ etc., are *metavariables* that stand for *actual* Boolean variables. As such, it is possible that, say, $\mathbf{p}$ and $\mathbf{q}$ stand for the same actual variable in some line of reasoning.

A proof that I ask you to write can be either Hilbert or Equational, UNLESS I ask for one of those styles specifically.

If so, the other proof style is worth 0 (F).

1. (5 MARKS)

Prove **Equationally** the associativity of $\wedge$. That is prove

$$\vdash ((A \wedge B) \wedge C) \equiv A \wedge (B \wedge C))$$

Use of Post's Theorem is NOT allowed (0 MARKS otherwise).

2. (5 MARKS) *True* or *False* CLAIM (below) **and WHY Exactly**?

I claim that statements —(1) and (2)— say the SAME THING:

$$\Gamma \vdash A \text{ iff } \Gamma \vdash B \tag{1}$$

$$\Gamma \vdash A \equiv B \tag{2}$$

HINT. If true, give a *proof*. If false, offer a *counterexample*. In the latter case you CANNOT and MAY NOT use *schemata*. Must use *specific* formulas A and B, and set Γ.

3. (3 MARKS) p and q are distinct variables.

I this correct? $p \vdash p \wedge q$. WHY EXACTLY?

4. (4 MARKS) Prove **Equationally** that $A, B \vdash A \equiv B$. Use of Post's Theorem is NOT allowed (0 MARKS otherwise).

In the following question the Deduction Theorem is recommended.

Use of Post's Theorem is NOT allowed in the following 4 Problems (0 MARKS otherwise).

5. (4 MARKS) Prove in **Hilbert-style** that

$$A \to B \vdash \neg B \to \neg A$$

6. (5 MARKS) Prove in **Hilbert-style** that

$$A \to B \vdash (B \to C) \to (A \to C)$$

7. (5 MARKS) Prove in **Hilbert-style** that

$$\vdash (A \to (B \to C)) \to (A \to B) \to (A \to C)$$

8. (5 MARKS) Prove in **Hilbert-style** that

$$\vdash (A \to (B \to C)) \to (B \to (A \to C))$$