Lassonde School of Engineering

Dept. of EECS

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MATH1090 B. Problem Set No2

Posted: Oct. 6, 2021

Due: Oct. 29, 2021; by 2:00pm, in eClass.

Q: **How do I submit?**

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

Post's Theorem use is not allowed in any question below.

1. (3 MARKS)

We proved in class that

$$\vdash A \equiv A$$

using the "*trick*" of a Leibniz "mouth"-variable **p** that does not appear in A.

Prove this again, Equationally, but without using this trick and without using Post's Theorem.

2. (5 MARKS) Prove Equationally that $A, B \vdash A \equiv B$.

3. (5 MARKS) Is Statement (1) below *True* or *False* **and WHY**?

$$\Gamma \vdash A \equiv B \text{ is equivalent to "}\Gamma \vdash A \textbf{ IFF } \Gamma \vdash B\text{"} \qquad (1)$$

Note that the sub-statement in quotes is a METAstatement. Note also that we have two "iff" in (1) above!

Caution. If a proof style is explicitly **required** in what follows, then any other style used gets 0 marks even if it is correct.

4. (5 MARKS) Prove **Equationally** that for any A and B

$$A, \neg A \vdash B \equiv \neg B$$

5. (4 MARKS) Prove **Equationally** that $\vdash A \to B \to A$.

6. (4 MARKS) Prove **Equationally** that $A \to B \vdash \neg B \to \neg A$.

7. (5 MARKS) Prove (choose your favourite: Equational or Hilbert proof) that $A \to B \vdash (B \to C) \to A \to C$.

8. Prove that $A \to B, C \to B \vdash (A \vee C) \to B$.

two proofs are required:

- (3 MARKS) One **with** the Deduction theorem (and a Hilbert-style proof; CUT rule allowed in this subquestion).
- (4 MARKS) One Equational, **WITHOUT** using the Deduction theorem.