Department of EECS

MATH1090 A. Problem Set No1

Posted: Sept. 17, 2022

Due: Oct. 4, 2022, by 2:00pm; in eClass.

Q: **How do I submit?**

A:

(1) **Submission must be ONLY ONE file**

(2) **Accepted File Types: PDF, RTF, MS WORD, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

1. (2 MARKS) Prove that the complexity of a *well-formed-formula* equals the number of its *left brackets*.

Hint. Analyse formula-constructions, or use induction on formulas.

2. (a) (3 MARKS) Prove that $(\bot)$ is *not* a wff.

(b) (2 MARKS) Why would Induction on complexity (or equivalently on formulas) be **not applicable** in this problem?

Hint. One way to address (a) is to analyse formula-constructions. The other is to look at the inductive definition of formulas, as it would be applied to "$(\bot)$".

3. (1 MARK) Prove that $(((q \to p) \to q) \to q)$ *is* a wff.

4. (6 MARKS) Recall that a **schema** is a tautology iff *all* its *instances* are tautologies.

Which of the following six schemata are tautologies? Show the whole process that led to your answers, *including truth tables or equivalent short cuts, if you used one or the other, and words of explanation.*

I note that in the six sub-questions below I am not using all the formally necessary brackets. You need to reinsert missing brackets to answer correctly.

Therefore be mindful of connective priorities and associativities!

- $A \to B \to (A \to \bot) \vee B$
- $(A \to B) \to (A \to \bot) \vee B$
- $A \wedge B \to A \equiv B$
- $A \to B \to \neg B \to \neg A$
- $(A \to B) \to \neg B \to \neg A$
- $A \wedge B \to A \to B$

5. (3 MARKS) Prove that *if* we have $A \models_{\text{taut}} \bot$, *then* we also have $A \models_{\text{taut}} B$ for every B.

6. (6 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.

In order to show that a tautological implication that involves *meta*-variables for formulas —i.e., it is a schema— is *incorrect* you *must* consider a special case that *is* incorrect (since some other special cases might *work*).

Show the whole process that led to each of your answers.

- $p \vee \neg p \models_{\text{taut}} \top$
- $\top \models_{\text{taut}} p \vee \neg p$
- $p \vee B \models_{\text{taut}} p$
- $A, B \to A \models_{\text{taut}} B$
- $A \equiv B \models_{\text{taut}} B \to A$
- $A \wedge B \models_{\text{taut}} B \vee A \equiv A \equiv B$

7. (6 MARKS) Write down the most simplified result of the following substitutions, *whenever the requested substitution makes sense*. Whenever a requested substitution does not make sense, explain exactly why it does not.

Show the whole process that led to each of your answers in each case.

Remember the priorities of the various connectives as well as that of the meta-expression "$[\mathbf{p} := \ldots]$"! The following formulas have not been written with all the formally required brackets.

- $(q \to p)[p := r]$
- $(q \to p)[p' := r]$
- $p \to \top[p := \bot]$
- $p \to \top[p := \mathbf{t}]$
- $(\bot \to r \to q)[q \wedge r := p]$
- $p \vee (q \wedge r)[p := q]$