Fast Gabor Filtering

Konstantinos G. Derpanis

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Gabor filtering represents a popular means for local signal analysis. Through convolution, Gabor filtering decomposes an input signal with inner products of a set of analysis functions in the form of a complex-modulated Gaussian window. In this note, we consider a fast algorithm for computing the Gabor filtering result proposed by Unser [3, 4]. Gabor filtering is performed indirectly by a series of three successive operations: (1) modulation (pointwise multiplication), low-pass filtering and demodulation (pointwise multiplication).

The standard convolution-based Gabor filtering process can be defined as follows,

$$\begin{aligned} I_1(x) &= I(x) * [w(x)\exp(-i\omega x)] && (1) \\ &= \sum_{k\in\mathbb{Z}} I(k)[w(k-x)\exp(-i\omega(k-x))], && (2) \end{aligned}$$

where $w(\cdot)$ represents a Gaussian window function, ω denotes the central frequency and $*$ symbolizes convolution.

The Gabor filter result (2) can be rewritten as follows (see Fig. 1 for a system diagram overview),

$$\begin{aligned} I_1(x) &= \sum_{k\in\mathbb{Z}} I(k)[w(k-x)\exp(-i\omega(k-x)] && (3) \\ &= \sum_{k\in\mathbb{Z}} I(k)[w(k-x)\exp(-i\omega k)\exp(i\omega x)] && (4) \\ &= \exp(i\omega x)\sum_{k\in\mathbb{Z}}[I(k)\exp(-i\omega k)]w(k-x) && (5) \\ &= \exp(i\omega x)\sum_{k\in\mathbb{Z}} I_0(k)w(k-x) && (6) \\ &= \exp(i\omega x)[I_0(x) * w(x)] && (7) \end{aligned}$$

where $I_0(x)$ is the premodulated input signal

$$I_0(x) = I(x)\exp(-i\omega x). \quad (8)$$

In the case where only the local power spectrum (or energy) of the signal is sought, the final demodulation step can be ignored. This follows from the fact that the demodulation step only contributes a phase shift to the intermediate result.

The efficiency of the modulation-based Gabor filtering process now depends on the manner in which the Gaussian filtering is realized. Possible approaches to speeding up the low-pass filtering step include the use of: separable Gaussians (in the case of multi-dimensional signals), binomial filters [2] and recursive filters (e.g., [1]).

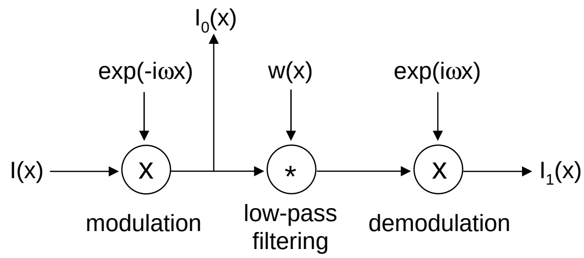

Figure 1: System diagram of modulation approach to Gabor filtering.

References

[1] R. Deriche. Recursively implementing the Gaussian and its derivatives. Technical Report 1893, Unite de recherche INRIA Sophia-Antipolis, 1993. Research Repport.

[2] B. Jähne. *Digital Image Processing, sixth edition.* Springer-Verlag, 2005.

[3] M. Unser. Comments on “A new approach to recursive Fourier transform”. *Proceedings of the IEEE*, 76(10):1395–1396, October 1988.

[4] M. Unser. Fast Gabor-like windowed Fourier and continuous wavelet transforms. *IEEE Signal Processing Letters*, 1:76–79, 1994.