Linear Dynamical Systems

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Linear dynamical systems (LDS) represent a generalization of hidden Markov models (HMM) (Rabiner, 1989) by allowing the underlying state-space to take on continuous values in $\mathbb{R}^n$ rather than a finite set of discrete symbols/states, as with HMMs. The simplest form of LDS is the first-order time-invariant Gauss-Markov process, given as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t-1) + \mathbf{v}, \quad \mathbf{v} \sim \mathcal{N}(0, \Sigma_v) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{w}, \qquad \mathbf{w} \sim \mathcal{N}(0, \Sigma_w), \tag{2}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ represents the underlying state vector at time, t, $\mathbf{y} \in \mathbb{R}^m$ is the observation vector with $m >> n$, $\mathbf{A}$ and $\mathbf{C}$ denote matrices that model linear processes and $\mathbf{v}$ and $\mathbf{w}$ are Gaussian noise processes with zero mean and covariances parameterized by the matrices $\mathbf{\Sigma_v}$ and $\mathbf{\Sigma_w}$, respectively. Similar to HMMs, LDSs are based on Markovian dynamic assumptions in the evolution of the system and conditional independence of the observations.

The model in (1) and (2) can be extended to model time-variant processes (i.e., non-stationary stochastic processes) by allowing the linear processes $\mathbf{A}$ and $\mathbf{C}$ to evolve as a function of time (i.e., $\mathbf{A}(t)$ and $\mathbf{C}(t)$). Note that the increased expressiveness of the time-variant model, introduces complexities in learning and inference.

A popular application of LDSs in computer vision lies in the context of dynamic textures. Dynamic textures (also known as temporal textures) are defined as a temporally continuous and infinitely varying stream of images that exhibit certain temporal statistics. LDSs have been used to model (e.g., (Saisan, Doretto, Wu & Soatto, 2001)), recognize (e.g., (Saisan, Doretto, Wu & Soatto, 2001)) and segment (e.g., (Doretto, Cremers, Favaro & Soatto, 2003; Chan & Vasconcelos, 2008)) dynamic textures.

References

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