Phase Constancy Constraint

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A common starting point for many optical flow estimation techniques (e.g. [3, 4]) is the brightness constancy constraint,

$$\nabla I \cdot (u, v, 1)^\top = 0 \tag{1}$$

where, $\nabla I = (I_x, I_y, I_t)^\top$ represents the gradient with respect to the spatial x, y and temporal parameter t of the image sequence $I(x, y, t)$. A more general approach is applying the constancy constraint on a prefiltered image that enhances certain structures, where the prefiltered image is derived from a local function of the image brightness. The locality property is important since it reduces the chances of the region under consideration spanning a motion discontinuity. For example, in [5] the authors prefilter the image with a set of oriented bandpass filters (i.e., first and second derivatives of a Gaussian) at multiple scales to enhance certain frequencies. The motivation for this particular prefiltering is that the approach may yield a more stable measurement matrix (and thus stable estimate). The source of the increased stability, though not guaranteed, is due to different parts of the spatial spectrum being sampled possibly resulting in a more widely spread out set of gradient directions. Another prefiltering example and the focus of this note is the phase constancy constraint [2], which consists of the application of the constancy constraint on a prefiltered image of local phase.

The initial step consists of extracting the local phase by convolving the image sequence by a set of complex bandpass filter (e.g., Gabor filter, nth derivative of a Gaussian) tuned to a narrow range of orientation, speed and scale; this strategy leverages the fact that the non-zero frequency components associated with the image volume (space plus time) of a translating pattern lies on a plane that includes the origin in the frequency domain. The filter response is $R(\mathbf{x}, t) = \rho(\mathbf{x}, t)e^{i\phi(\mathbf{x},t)}$, where $R(\mathbf{x}, t)$ denotes a measure of the local amplitude of the image and $\phi(\mathbf{x}, t)$ denotes the local phase.

Next, the constancy constraint is applied to the phase component,

$$\nabla_{\mathbf{x}}\phi(\mathbf{x}, t) \cdot (u, v)^\top + \phi_t(\mathbf{x}, t) = 0 \tag{2}$$

where $\nabla_{\mathbf{x}}\phi$ can be calculated using the following identity,

$$\nabla_{\mathbf{x}}\phi = \frac{\mathrm{Im}[R^*(\mathbf{x}, t)\nabla_{\mathbf{x}} R(\mathbf{x}, t)]}{R^*(\mathbf{x}, t)R(\mathbf{x}, t)} \tag{3}$$

and $R^*(\cdot)$ denotes complex conjugate. Like the brightness constancy constraint, we still face the aperture problem when considering the output of a single filter at a point. Possible solutions include considering the phase outputs from multiple orientations and/or the considering the phase within a local patch.

An advantage of the phase-based approach is that it is relatively insensitive to variations in illumination, contrast and perspective projection deformations. Finally, in a comparison of popular optical flow techniques cited in the literature, the phase-based approach was found to produce the most accurate results on a test suite of synthetic and real-world image sequences [1].

References

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